# Investigation of Chemical Reaction on Micropolar Fluid over an Exponentially Stretching Sheet

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#### Abstract

Investigation on steady boundary layer flow of heat and mass transfer of a micro polar fluid over an exponentially stretching sheet has been made in this paper. The governing mathematical model of the problem considered basically. The basic partial differential equations are reduced to a system of nonlinear ordinary differential equations by using similarity approach which are solved analytically using perturbation technique and numerically by Mathematica. Numerical calculations for the analytical expressions are carried out and the results are shown graphically. The effects of the various dimensionless parameters related to the problem on the velocity, angular velocity, temperature and concentration fields are discussed. Comparative analysis has been made for both the numerical and perturbation solution and found a very good agreement.

**Keywords:** Boundary Layer Flow, Heat Transfer, Mass transfer, Micropolar Fluid, Exponential Stretching sheet, Perturbation Method.

# **1. Introduction**

It is well known that the fluids are classified into Newtonian fluid and non-Newtonian fluid. Many researchers have done research in Newtonian fluid but for the non-Newtonian fluid, which has viscosity change when forces are applied. A good example of non-Newtonian fluid is a micro polar fluid. The micro polar fluid is a fluid with microstructure. Micro polar fluid consists of rigid, randomly oriented particles with their own micro rotations, suspended in a viscous medium. The heat transfer analysis, for steady boundary layer stagnation point flow of a micropolar fluid over an exponentially stretching sheet and the analytical solution was given by Abdul Rehman[1]. Adhikari and Maiti[2], investigated about a steady two dimensional in compressible magnetohydrodynamics micropolar fluid flow

towards a stretching or shrinking vertical sheet. Afifi.et.al[3], discussed about the influence of magnetic field on wall properties for peristaltic motion of micropolar fluid in a flexible tubes. Agarwal.et.al [5] had given finite element solution of flow and heat transfer of a micropolar fluid over a stretching sheet. The micropolar fluid theory and its application to low concentration suspension flow was discussed by Ahmadi[6].

The heat transfer process in boundary layer flow of micropolar fluid over an exponentially permeable shrinking sheet was analyzed by Auranzzaib[7]. The unsteady laminar flow of an incompressible micropolar fluid over a stretching sheet was investigated by Bachok[8].Bhargava[8] had given numerical solution of free convective micropolar fluid flow through two parallel porous vertical plate. Don[9] had studied the transient heat and mass transfer of micropolar fluid between porous vertical channel with third kind of boundary condition. An analytical study had been done for unsteady hydromagnetic heat mass transfer for a micropolar fluid bounded by semi-infinite vertical permeable plate, Dulalpal[10]. An asymptotic solution is developed for large distance away from the leading edge is determined to the steady free convection from a vertical isothermal plate in a strong cross magnetic field and immersed in а micropolar fluid. Hassan Waqas[11] has studied about the MHD forced convective flow of micropolar fluid past a moving boundary surface with prescribed heat flux and radiation. The flow of an incompressible constant density micropolar fluid past a porous stretching sheet was investigated by Heruska[14].

The steady laminar MHD boundary layer flow past a wedge immersed in an incompressible micropolar fluid in a presence e of a variable

field investigated magnetic was by Ishak[15].Network stimulation method was used to solve the transient problem of coupled heat and mass transfer of micropolar fluid in a magneto hydrodynamic free convection by Joaquin Zueco[16]. The influence of first order chemical reaction and thermal radiation on hydrodynamic free convection heat and mass transfer of a micropolar fluid, obtained by Das[17]. Perturbation approximation was used to give analytical solution. The numerical study of hydrodynamic (MHD) flow and heat transfer characteristic of a viscous incompressible electrically conducting micropolar fluid had been reported by KashifAli[18].OHAM and numerical results are observer with excellent agreement for injective micropolar flow in a porous channel was reported by Kelson[19]. The perturbation technique was worked to solve the magnetic field effect on a free convective mass transfer flow of chemical reactive micropolar fluid over an vertical porous plate, Khan EnaetHossain[20].Khilapsingh [21], investigated about the heat and mass transfer characteristic of the free convective flow on a vertical plate in porous media with variable wall temperature and concentration in a doubly stratified and viscous dissipating micropolar fluid with chemical reaction, heat generation and ohmic heating. Lakshmi narayana[22], studied about unsteady two dimensional mixed convective flow of an viscous incompressible electrically conducting micropolar fluid in presence of second order slip flow. The

numerical study on effect of soret and Non-uniform heat source on MHD non-darcian convective flow over a stretching sheet in a dissipative micropolar fluid with radiation, reported by Mabood[23]. Non uniform heat source sink and soret effect on MHD non darcian convective flow past a stretching sheet in a micropolar fluid with radiation was studied by Mabood and Ibrahim[24]. In the presence of cattaneochristov heat flux, the convective micropolar fluid flow past over a non linear stretching surface, Ganeswarareddy[25].Free convective micropolar fluid flow and heat transfer over a shrinking sheet with heat source, reported by Mishra[26]. A mathematical stimulation to the cooling process of a flat moving surface using a weak concentration micropolarnanofluid as a cooling medium had been investigated by Mohamed[27].

A boundary layer analysis study is presented to study the effects of magnetic field with vectored surface mass transfer and induced buoyancy streamwise pressure gradient on heat transfer to a

horizontal plate placed in a micropolar fluid is by Mohammadein[28].Numerical investigated solution of two dimensional stagnation flows of micropolar fluid towards a shrinking sheet by using SOR Iterative procedure is studied by Mohammed Shafique[29]. SajjadHussain[30] has studied about the MHD flow of micropolar fluids over a shrinking sheet with mass transfer. Olajuwon[31] have investigated about the effect of thermo-diffusion and thermal radiation on unsteady heat and mass transfer of free convective by a semi infinite porous plate in a rotating frame under the action of transverse magnetic field with suction.Ramzan[32] discussed about the flow of mciropolarfluis past a permeable stretching sheet in attendance of joule heating, thermalradiation, partial slip and MHD with convective boundary condition. The differential transform method is applied to the micropolar flow in a porous channel with mass injection is reported by Rashidi[33]. MHD flow of micropolar upper convectedMaxwell (UCM) fluid with hyperbolic heat flux is studied by sabeel Khan[34]. Sajjid Hussain[35] has investigated about the flow in MHD stagnation and heat transfer for micropolar fluid towards a permeable cylinder with moving boundaries. The flow of micropolar fluid towards a porous stretching surface was studied by shamila Khalid[36]. The heat and mass transfer flow of Magneto hydrodynamic micropolar fluid in the presence of viscous dissipation and chemical reaction, sivareddysheri[37]. Srinivas[38] examined in the flow and heat transfer of entropy generation for inclined channel of two immiscible an micropolarfluid. MHD convective flow and heat transfer of a micropolar fluid past a continuously moving vertical porous plate in the presence of heat generation/absorption with constant suction has been analysed with constant suction has been analyzed numerically by Rahman[39]. XinHuisi[40] had given a analytical solution to the micropolar fluid flow through semi porous with expanding or contracting wall.

This article is concerned with the steady MHD boundary layer flow of heat and mass transfer of a micro polar fluid flowing over an exponentially stretching sheet. The solution of the problem is obtained numerically by Mathematica Software and the Partial differential equation is solved by perturbation method. Comparisons are made for both the solution.

(1)

# 2. Mathematical Formulation

Let us consider the steady two dimensional incompressible flows of heat and mass transfer on a micro polar fluid over a exponentially permeable shrinking sheet. It is assumed that the shrinking velocity is in exponential form and it is taken as  $U_w = ae^{x/L}$  with a>0 which is a shrinking constant. Using the boundary layer approximation, the equation of motion for the micro polar fluid, heat and mass transfer may be written as:

## The governing equations are:

## **Continuity Equation:**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

**Momentum Equation:** 

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = U_{\infty}\frac{dU_{\infty}}{dx} + (v + \frac{k}{\rho})\frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho}\frac{\partial u}{\partial y}$$
(2)
Angular momentum Equation:

Angular momentum Equation:

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial x} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial x^2} - \frac{k}{\rho j}(2N + \frac{\partial u}{\partial y})$$
(3)

**Energy Equation**:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial x} = \frac{k^*}{\rho c_p}\frac{\partial^2 T}{\partial y^2}$$
(4)

**Concentration Equation:** 

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D_m \frac{\partial^2 c}{\partial y^2} - K_r (C - C_\infty)^n$$
(5)

Subject to the boundary condition

$$u = U_w, v = 0, N = n(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}), T = T_w(x), C = C_w(x)aty = 0$$

$$u \to U_{\infty}, N \to 0, T \to T_{\infty}, C \to C_{\infty} asy \to \infty$$
 (6)

where u and v are velocity components in x and y directions respectively,  $\boldsymbol{v}$  is the kinematic viscosity,  $\boldsymbol{\mu}$ is the dynamic viscosity,  $\rho$  is the density, N is the microrotation, j is the micro inertia per unit mass,  $\gamma$ is the spin gradient viscosity, k is the vortex viscosity, T is the temperature, L is the reference length,  $k^*$  is the thermal conductivity of the fluid,  $c_n$ is the specific heat,  $T_0$  being a constant which measures the rate of temperature increment along the sheet and  $T_{\infty}$  is the free stream temperature assumed

to be constant. It is well Known that n=0 is called a strong concentrate particle which flows close to the micro-elements to the wall surface that unable to rotate. For n=0.5 the anti-symmetric part of the stress tensor vanishes and denote weak concentrations. whereas n=1 is used for modelling of turbulent boundary layer flows.

Here for the exponential stretching sheet the expression for  $U_{\infty}$ ,  $U_{w}$ ,  $T_{w}$  and  $C_{w}$  are defined as

$$U_{\infty} = ae^{\frac{x}{L}}, U_{W} = be^{\frac{x}{L}}, T_{w} = T_{\infty} + ce^{\frac{x}{L}}, C_{w} = C_{\infty} + de^{\frac{x}{L}}$$
(7)

The Cauchy Riemann equations

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}$$
(8)

where  $\psi(x,y)$  is the stream function.

Now, the following transformation is introduced.

$$u = ae^{\frac{\lambda}{L}}f'(\eta), v = -(\frac{\upsilon a}{2L})^{\frac{1}{2}}e^{\frac{\lambda}{2L}}(f(\eta) + \eta f'(\eta)), N = a(\frac{a}{2\upsilon L})^{\frac{1}{2}}e^{\frac{\lambda a}{2L}}M(\eta)$$

(9)

$$\theta = \frac{T - T_{\infty}}{T_{W} - T_{\infty}}, \eta = \left(\frac{a}{2\nu L}\right)^{\frac{1}{2}} e^{\frac{x}{2L}} y, \varphi = \frac{C - C_{\infty}}{C_{W} - C_{\infty}}$$

Using eq(8),the nonlinear partial differential equation(2) to (5) are transformation into the following ordinary differential equation:

$$f^{+} + \frac{1}{1+K} (ff^{+} - 2f^{+} + 2) + \frac{K}{1+K} M^{+} = 0$$
(10)  
$$M^{+} + \frac{1}{\Lambda} (fM^{+} - 3f^{+}M) - \frac{K\chi}{\Lambda \operatorname{Re}} (2M + f^{+}) = 0$$
(11)

$$\theta' + \Pr(f\theta' - 2f'\theta) = 0$$
(12)

$$\varphi - 2Scf \varphi + Scf \varphi - 2Sc\gamma \varphi^n = 0 \tag{13}$$

The boundary conditions are

$$f(0) = 0, f'(0) = \varepsilon, f' \to 1as\eta \to \infty$$
  

$$M(0) = -nf'(0), M \to 0as\eta \to \infty$$
  

$$\theta(0) = 1, \theta \to 0as\eta \to \infty$$
  

$$\varphi(0) = 1, \varphi \to 0as\eta \to \infty$$
(14)

Where prime denote differentiation with respect to

$$\eta$$
,  $K = \frac{k}{\mu}$  is the material parameter,  $\Pr = \frac{v}{\alpha}$  is the prandtl number,  $\Pr = \frac{LU_x}{2v}$  is the Non-similar Reynolds number,  $S = \frac{v_0}{\sqrt{\frac{va}{2L}}}$  is the mass suction/injection

parameter with S>0 for mass suction parameter and S<0 for mass injection parameter,  $\underline{v}_{=Sc}$  is Schmidt  $\overline{D_m}$ 

 $\gamma = \frac{K_r}{a} C_0^{n-1} e^{(n-2)\frac{x}{L}} \mathbf{i} \mathbf{S}$ chemical reaction number, parameter.

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The quantities of physical interest are the local skin friction coefficient, the local couple stress and the local Nusselt number and those are defined as

$$C_{f} = \frac{\left[(\mu + k)\frac{\partial u}{\partial y} + kN\right]_{y=0}}{\rho U_{w}^{2}}$$

$$M_{x} = -\frac{\gamma(\frac{\partial N}{\partial y})_{y=0}}{\rho x U_{w}^{2}}$$

$$Nu_{x} = -\frac{x}{(T_{w} - T_{\infty})} \left(\frac{\partial T}{\partial y}\right)_{y=0}$$
(15)

that is,

$$\operatorname{Re}_{x}^{1/2} C_{f} = \frac{1}{\varepsilon^{2}} (1 + (1 - n)K) f^{"}(0)$$
  

$$M_{x} \operatorname{Re}_{x} = (1 + \frac{K}{2})M^{'}(0)$$
  

$$Nu_{x} \operatorname{Re}_{x}^{-1/2} = -\theta^{'}(0)$$
(16)

## 3. Perturbation Method

The results are obtained from an analytical solution of the problem with perturbation techniques

$$\begin{aligned} \zeta &= \eta f_w \\ f(\eta) &= f_w G(\xi) \\ M(\eta) &= f_w^3 H(\xi) \\ \theta(\eta) &= f_w^3 X(\xi) \\ \varphi(\eta) &= f_w^4 \chi(\xi) \end{aligned}$$
(17)  
Now the model with small quantity,  
$$(1+K)G^{"} + (GG^{"} - 2G^{"} + \varepsilon^2 2) - KM^{"} = 0 \\ H^{"} + \frac{1}{\Lambda} (GH^{"} - 3G^{'}H) + \frac{K\zeta}{\Lambda \text{Re}} (2H + G^{"}) = 0 \\ S^{"} + \Pr(GS^{"} - 2G^{'}S) = 0 \\ \chi^{"} - 2ScG^{'}\chi + G\chi^{'} - 2Sc\gamma\varepsilon\chi = 0 \\ \chi^{"} - 2ScG^{'}\chi + G\chi^{'} - 2Sc\gamma\varepsilon\chi = 0 \end{aligned}$$
(18)  
And the boundary conditions are  
$$G(0) = 0, G^{"} = \varepsilon^2, G^{"} = \varepsilon, \quad at \quad \xi \to \infty \end{aligned}$$

$$\begin{aligned} G(0) &= 0, \ G = \varepsilon, \ G = \varepsilon & at \quad \zeta \to \infty \\ H(0) &= -nG(0), \ H(0) = 0 & at \quad \zeta \to \infty \\ S(0) &= \varepsilon, S = 0 & at \quad \zeta \to \infty \\ \chi(0) &= \varepsilon^2, \ \chi \to 0 & as \quad \zeta \to \infty \end{aligned}$$
(19)

Now for the large suction  $(f_w > 0) \xi$  will be small.

Therefore following G,H and S can be expanded in terms of the small perturbation quantity  $\mathcal{E}$ 

$$G(\xi) = 1 + \varepsilon G_{1}(\xi) + \varepsilon^{2}G_{2}(\xi) + \varepsilon^{3}G_{3}(\xi) + \dots - H(\xi) = \varepsilon H_{1}(\xi) + \varepsilon^{2}H_{2}(\xi) + \varepsilon^{3}H_{3}(\xi) + \dots - S(\xi) = \varepsilon S_{1}(\xi) + \varepsilon^{2}S_{2}(\xi) + \varepsilon^{3}S_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{2}\chi_{2}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon^{3}\chi_{3}(\xi) + \dots - \chi(\xi) = \varepsilon \chi_{1}(\xi) + \varepsilon \chi_{1}(\xi)$$

The dimensionless equation (9) to (12) is transform into the following first and second order equations,

$$(1+K)G_{1}^{"} + G_{1}^{"} - KH_{1}^{'} = 0$$
  

$$H_{1}^{"} + \frac{1}{\Lambda}H_{1}^{'} = 0$$
  

$$S_{1}^{"} + \Pr S_{1}^{'} = 0$$
  

$$\chi_{1}^{'} + Sc\chi_{1}^{'} = 0$$
(21)

The boundary conditions are

$$G_{1} = -\frac{1}{\varepsilon}, G_{1}^{'} = \varepsilon, G_{1}^{'} = 0at\xi \to \infty$$

$$H_{1} = 0, H_{1} = 0at\xi \to \infty$$

$$S_{1} = 1, S_{1} = 0at\xi \to \infty$$

$$\chi_{1}(0) = \varepsilon, \chi_{1}(0) = 0at\xi \to \infty$$
(22)

$$(1+K)G_{2}^{"}+G_{2}^{'}+G_{1}G_{1}^{'}-2G_{1}^{1'}+2-KH_{2}^{'}=0$$

$$H_{2}^{'}+\frac{1}{\Lambda}(H_{2}^{'}+G_{1}H_{1}^{'}-3G_{1}H_{1})+\frac{K\zeta}{\Lambda \operatorname{Re}}(2H_{1}+G_{1}^{'})=0$$

$$S_{2}^{'}+\operatorname{Pr}(S_{2}^{'}+G_{1}S_{1}^{'}-2S_{1}G_{1}^{'})=0$$

$$\chi_{2}^{'}+Sc\chi_{2}^{'}-2ScG_{1}\chi_{1}+ScG_{1}\chi_{2}^{'}-2Sc\gamma\chi_{1}=0$$
(23)

The boundary conditions are

$$G_{2} = 0, G_{2} = 0 \qquad at \quad \xi \to 0, \ G_{2} = \frac{1}{\epsilon} \qquad at \quad \xi \to \infty$$

$$H_{2} = 0 \qquad at \quad \xi \to 0, \ H_{2} = 0 \qquad at \quad \xi \to \infty$$

$$S_{2} = -\frac{1}{\epsilon} \qquad at \quad \xi \to 0, \ S_{2} = 0 \qquad at \quad \xi \to \infty$$

$$\chi_{2} = 0 \qquad at \quad \xi \to 0, \ \chi_{2} = 0 \qquad at \quad \xi \to \infty$$

$$(24)$$

The solution of first and second order equations are

$$G_{1} = -\frac{1}{\varepsilon} - \frac{\varepsilon}{A_{2}} + \frac{\varepsilon}{A_{2}} e^{-A_{2}\xi}$$

$$H_{1} = 0, S_{1} = e^{-\Pr\xi}, \chi_{1} = \varepsilon e^{-Sc\xi}$$

$$(25)$$

$$G_{2} = (-\frac{1}{\varepsilon A_{2}} + 2A_{7} - 3A_{8} + 2A_{9} + 3A_{10} - A_{11}) + \frac{\varepsilon}{\varepsilon} + (\frac{1}{\varepsilon A_{2}} - A_{7} + 2A_{8} - A_{9} - 2A_{10} - A_{11})e^{-A_{2}\xi}$$

$$H_{2} = A_{6} e^{-A_{2}\xi}$$

$$H_{2} = A_{6} e^{-A_{2}\xi}$$

$$g_{2} = -A_{2} e^{-Pr\xi} - A_{4} e^{(A_{2} - Pr)\xi}$$

$$\chi_{2} = -(A_{12} + A_{13} + A_{14} + A_{15})e^{-Sc\xi} + A_{12} e^{-Sc\xi} + A_{13} e^{-Sc\xi} + A_{14} e^{-(A_{2} + Sc)\xi} + A_{15} e^{-Sc\xi}$$

$$(26)$$

The velocity Equation is

$$\begin{split} f &= f_w G(\xi) \\ &= f_w (1 + \varepsilon (-\frac{1}{\varepsilon} - \frac{\varepsilon}{A_2} + \frac{\varepsilon}{A_2} e^{-A_5\xi}) + \varepsilon^2 (-\frac{1}{\varepsilon A_2} + \frac{\xi}{\varepsilon} + \frac{1}{\varepsilon A_2} e^{-A_5\xi} - A_7 e^{-A_5\xi} + A_8 e^{-2A_5\xi} - A_9 e^{-A_5\xi} + A_9 e^{-2A_5\xi} - A_9 e^{-A_5\xi} + \xi^2 - A_{10} e^{-A_5\xi}) \end{split}$$

Angular Momentum Equation is

$$M = f_w^3 H(\xi) = f_w^3 \varepsilon^3 A_6 e^{-A_2 \xi}$$
  
Energy Equation is  
$$\theta = f_w^2 H(\xi) = f_w^2 (\varepsilon e^{-\Pr\xi} + \varepsilon^2 (A_5 e^{-\Pr\xi} - A_4 e^{(A_2 - \Pr)\xi}))$$

Concentration Equation is

$$\begin{split} \chi(\eta) &= f_w^4 \chi(\xi) = f_w^4 (\varepsilon \chi_1(\xi) + \varepsilon^2 \chi_1(\xi)) \\ &= f_w^4 (\varepsilon (\varepsilon e^{-Sc\xi}) + \varepsilon^2 (-(A_{12} + A_{13} + A_{14} + A_{15}) e^{-Sc\xi} + A_{12} e^{-Sc\xi} + A_{13} e^{-Sc\xi} + A_{14} e^{-(A_2 + Sc)\xi} + A_{15} e^{-Sc\xi})] \end{split}$$

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(20)

# 4. Numerical Solution

The set of couple boundary layer equations (10) - (14) subject to boundary conditions (15) has been solved by using NDsolve in Mathematica. The validity of the numerical computations has been confirmed via benchmarking with results of perturbation method. The effects of the various dimensionless parameters related to the problem on the velocity, angular velocity, temperature and concentration fields are discussed with the help of graphs. Comparative analysis has been made for both the numerical and perturbation solution and found a very good agreement.

## 5. Results and Discussions

In this study, we described the role of physical constraints on fluid velocity, microrotation, temperature and concentration. Numerical values are expressed in terms of graphs to analyze the impact of various flow controlling parameters like Micro polar parameter, Micro polar co-efficient, stretching ratio parameter, Prandtl number, non-similar Reynolds number, Schmidt number and chemical reaction parameter. The comparison between the obtained results by numerical and analytical solutions is presented in the figures 1–12.

Figures.1 & 2 describe the effect of the Micro polar parameter (K) and Stretching ratio parameter ( $\varepsilon$ ) on the velocity profile. Evidently, there is a decrement function in the velocity profile as K increases due to the fact thickening of the boundary layer is also reduced. But the velocity profiles rise with an increase in Stretching ratio parameter ( $\varepsilon$ ).

The effect of Micro polar co-efficient (  $\Lambda$  ) and Stretching ratio parameter ( $\epsilon$ ) on the Micro rotation velocity profile M by numerical and analytical solutions are presented in Figures.3 & 4. An implement in the Micro polar parameter (  $\Lambda$  ) reduces the Microrotation velocity profiles. Also, an increase in  $\epsilon$  enhances the Micro rotation velocity profile.

Figure 5 & 6 demonstrated the effect Micro polar parameter (K) and non-similar Reynolds number (Re) on Micro rotation velocity profiles M numerically and analytically. Increasing Micro polar parameter increase Microrotation profile which assist the flow to thicken the boundary layer and increasing non-similar Reynolds number decrease the Microrotation profile. The influence of Prandtl Number (Pr) on the temperature profile by numerical and analytical solutions is displayed in Figure.7. It is observed from the graph that the temperature decreases with increasing values of Prandtl number (Pr). Figure.8 depicts the micro polar parameter (K) on temperature profile. It is evident that with an incrementing function in K increase temperature profile

In Figure 9, concentration profile indicates an diminishing trend for higher values of Schmidt number(Sc) . Figure.10 preserves the impact of chemical reaction parameter ( $\gamma$ ) effect. It has been found that larger value of chemical reaction parameter ( $\gamma$ ) leads to stronger the concentration field and less boundary layer thickness.







#### 6. Conclusion

The effects of the various dimensionless parameters related to the problem on the velocity, angular velocity, temperature and concentration fields are discussed with the help of graphs. Comparative analysis has been made for both the numerical and perturbation solution and found a very good agreement. The conclusions are as follows:

- Effect of the Micro polar parameter (K) and Stretching ratio parameter (ε) on the velocity profiles increases due to the fact thickening of the boundary layer is also reduced. But the velocity profiles rise with an increase in Stretching ratio parameter (ε).
- The effect of Micro polar co-effienct(Λ) and Stretching ratio parameter (ε) on the Micro rotation velocity profile M by numerical and analytical solutions are presented. An implement in the Micro polar parameter (Λ) reduces the Microrotation velocity profiles. Also, an increase in ε enhances the Micro rotation velocity profile.
- Increasing Micro polar parameter increase Microrotation profile which assist the flow to thicken the boundary layer and increasing non-similar Reynolds number decrease the Microrotation profile.
- Temperature decreases with increasing values of Prandtl number (Pr). Figure.8 depicts the micro polar parameter (K) on temperature profile. It is evident that with an incrementing function in K increase temperature profile
- Concentration profile indicates an diminishing trend for higher values of Schmidt number(Sc). It has been found that larger value of chemical reaction parameter (γ) leads to stronger the concentration field and less boundary layer thickness.

Appendix

$$A_{1} = -\frac{1}{\Lambda}, A_{2} = \frac{1}{1+K}, A_{3} = \frac{\varepsilon \operatorname{Pr}}{A_{2}} + 2\varepsilon$$
$$A_{4} = \frac{A_{3}}{A_{2}^{2} - 2A_{2}\operatorname{Pr} + \operatorname{Pr}A_{2}} A_{5} = -\frac{1}{\varepsilon} + A_{4}$$
$$A_{6} = \frac{K\xi^{2}\varepsilon\chi}{\Lambda\operatorname{Re}A_{2}} A_{7} = \frac{\xi^{2}}{A_{2}^{3}(1+(1+K)A_{2})},$$

$$A_{9} = \frac{\xi^{2} \varepsilon^{2}}{A_{2}^{4} (1 + (1 + K)A_{2})},$$

$$A_{10} = \frac{2\varepsilon^{2}}{4A_{2}^{2} (1 + (1 + K)2A_{2})},$$

$$A_{11} = \frac{KA_{2}A_{5}}{A_{2}^{2} (1 + (1 + K)A_{2})},$$

$$A_{12} = \frac{-\xi}{2Sc}, A_{13} = \frac{Sc\varepsilon^{2}\xi}{2A_{2}},$$

$$A_{14} = \frac{-2Sc\varepsilon^{2}}{A_{2}(A_{2} + Sc)}, A_{15} = 2\gamma\varepsilon\xi$$

# References

- Abdul Rahman., Naveed Sheikh., (2017)Boundary layer Stagnation point flow of Micropolar Fluid over an exponentially Stretching Sheet. International Journal Fluid Mechanics and Thermal Sciences.vol 3: Issue 3, pp 25-31.
- [2] Adhikari.A and Maitti.A.K (2014), MHD micropolar fluid flow towards a vertical surface in presence of heat source/sink under radiative heat flux, Jr.ofIntmath.virt.Inst (J.of IMVI) vol 4,pp 1-25.
- [3] Afifi NA, Mahmoud SR, Al-Isede HM. (2011): Effect of magnetic field and wall properties on peristaltic motion of micropolar fluid,International Mathematical Forum, 6 (27):1345-1356.

- [4] Agarwal R.S., Rama B. and Balaji A.V.S. (1989): Finite element solution of flow and heat transfer of a micropolar fluid a over stretching sheet. - Int. Jr. Engg. Sci., vol.2, pp.1421-1428.
- [5] Ahmadi, G. (2013). Self-similar solution of incompressible micropolar boundary layer flow over a semi-infinite plate, Int. J. Eng. Sci., 14, 639-646.
- [6] Aurangazaib,Md.SharifUddin,KrishnenduBhatta charyya,SharidanShafie,(2016) Micropolar fluid flow and heat transfer over an exponentially permeable shrinking sheet, Propulsion and power Research,Vol 5, Issue 4,pp 310-317. ttp://dx.doi.org/10.1016/j.jppr.2016.11.005.
- [7] Bachok.N,Ishak.A and Nazar.R.,(2010), Flow and heat transfer over an unsteady strtching sheet in a micropolar fluid with prescribed heat flux, Int.Jr.of mathematical Models and Methods in applied sciences,vol 3,Isssue 4,pp 167-176.
- [8] Bhargava R., Kumar L. and TakharH.S. (2003): Numerical solution of free convection MHD micropolar fluid flow between two parallel porous vertical plates. – Int. J. Eng. Sci., vol.41, pp.123-136.
- [9] Doh.D.H, Muthatamilselvan.M,Prakash.D(2016) Transient heat and mass transfer of micropolar fluid between porous vertical channel with boundary conditions of third kind, Int.Jr. of Nonlinear sciences and numerical simulation,vol 117,Issue 5.
- [10] Dulal, Babulal Talukdar., (2012), Pertubation techni que for unsteady MHD mixed convection periodic flow, heat and mass transfer in micropolar fluid with chemical reaction in the presence of thermal radiation, Central European Jr of Physics, vol 10, Issue 5, pp 1150-1167.
- [11] Gorla R.S.R., Takhar H.S. and Slaouti A. (1998): Magnetohydrodynamic free convection boundary layer flow of a thermomicropolar fluid over a vertical plate. – Int. J. Eng. Sci., vol.36, No.3, pp.315-327.
- [12] Hassan W, Sajjad H, Humaira S, Shanila K. MHD forced convective flow micropolar fluid past a moving boundary surface with prescribed heat flux and radiation. British Journal of Mathematics & Computer Science. 2017;21(1):1-14. 13.
- [13] Hassanien I.A. and Gorla R.S.R.(1990): MHD forced convective flow micropolar fluid past a moving boundary surface with prescribed heat flux and radiation. - Acta Mech., vol.84, pp.191-199.

- [14] Heruska M.H., Watson L.T. and Sankara K.K. (1986): Micropolar flow past a porous stretching sheet. - Comp. Fluids, vol.14, pp.117-129.
- [15] Ishak A., Nazar R. and Pop I. (2008): MHD boundary-layer flow of a micropolar fluid past a wedge with variable wall temperature. – Acta Mech., vol.196, pp.75-86.
- [16] Joaquin Zueco (2007), Transient free convection with mass transfer MHD micropolar fluid in a porous plate by the network method, Numerical method of fluids, vol57,Issue 7, pp 861-876.
- [17] Kalidas das,(2011), Effect of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid in a rotating frame of reference,Int.Jr. of heat and mass transfer,vol 54, No 15-16,pp 3505-3513.
- [18] Kashif A, Muhammad A.(2014) Numerical simulation of the micropolar fluid flow and transfer in a channel with a shrinking and a stationary wall. Journal of Theoretical and Applied Mechanics; 52(2):557-567.
- [19] Kelson.N.A., Desseaux.A., Farrell.T.W., (2003) Micropolar flow in a porous channel with high mass transfer, ANZIAM J.44(E), pp 479-495.
- [20] Khan.EnaetHossain, Maruf Hassan .S.M,SyedAuntashireRahman and Md.MohidulHaque (2017), Natural convective mass transfer MHD flow of chemically reactive micropolar fluid pass a vertical porous plate, Physical science Int. Jr, vol13,Issue 1.
- [21] Khilap, S., Manoj, K. (2015). Effect of viscous dissipation on double stratified MHD free convection in micropolar fluid flow in porous media with chemical reaction, heat generation and ohmic Heating, Chemical and Process Engineering Research, 31, 75-80.
- [22] Lakshmi Narayan.K and Gangdhar.K (2015), Second order slip flow of the MHD Micropolar fluid over an unsteady stretching surface, Advance in applied science research, vol 6,Issue 8,pp 224-241.
- [23] Mabood.F and Ibrahim.S (2016), Effect of soret and Non-uniform heat source of MHD nondarician convective flow over a stretching sheet in a dissipative micropolar fluid with radiation,Jr.Applied fluid mechanics,vol 9,Issue 5,pp 2503-2513.
- [24] Mabood.F, Ibrahim.S.M, Rashidi.M.M, Shadloo.M.S, Giulio.Lovenziniv (2016), Non uniform heat source.sink and soret effect on MHD non Darician convective flow past a stretching sheet in a micropolar fluid with

radiation, ,Int.Jr.of heat and mass transfer,vol 93,pp 674-682.

- [25] Machireddy ., Gnaneswara Reddy., Gorla Rama subba Reddy., (2017)Micropolar fluid flow over a Non linear stretching convectively heated vertical surface in the presence of Cattaneo-Christove heat flux and Viscous Dissipation. Frontiers in Heat and Mass Transfer vol 8: 20: pp 1-9.
- [26] Mishra.S.R.,Khan.I.,Al-Mdallal.Q.M.,(2018) Free convective micropolar fluid flow and heat transfer over a shrinking sheet with heat source,Case studies in thermal Engg,vol 11, pp 13-119.
- [27] Mohamed S.Abdel-Wahed.,(2017)Flow and heat transfer of a weak concentration micropolarnanofluid over steady.unsteady moving surface,Applied Physics A,123:195.
- [28] Mohammadein A.A. and Gorla R.S.R. (1996): Effects of transverse magnetic field on mixed convection in a micropolar fluid on a horizontal plate with vectored mass transfer. – Acta Mech., vol.118, pp.1-12.
- [29] Mohammed Shafique, AtifNazir, Fatima Abbas., (2015) Numerical solution of two dimensional stagnation flows of micropolar fluid towards a shrinking sheet by using SOR Iterative procedure, Jr. of progressive research in mathematics, vol 3, Issue 1.
- [30] Mohammed Shafique, FatimaAbbas, abdur Rashid., (2013) MHD Viscous flow of Micropolar fluid due to a shrinking sheet, Int.Jr.of Emerging technology and Advanced Engineering, vol 3, Issue 7.
- [31] Olanjuwon.B.I,Oahimire.J.I.,(2013), Unsteady free convection heat and mass transfer in an MHD Micropolar fluid in the presence of thermo diffusion and themalradiation,Int.Jr. of pure and Applied mathematics,vol 84, No 2,pp 15-37.
- [32] Ramzan.M,Farooq.M,Hayat.T,Jae Dong (2016), Radiative and Joule heating effects in the MHD flow of a micropolar fluid with partial slip and convective boundary conditions, Jr.of Molecular liquids, vol 221,pp 394-400.
- [33] Rashidi.M.M, Mohimanianpour.S.A, Laroqi.N (2010), A semi analytical solution of micropolar flow in a porous channel with mass injection by using differential transform method, Nonlinear:Analysis: Modelling and control, vol 15, No 3,pp 341-350.
- [34] Sabeel M. Khan, Hammad. M, Batool. S and Kaneez. H (2017), Investigation of MHD effect

and heat transfer for the upper-convected Maxwell(UCM-M) micropolar fluid with Joule heating and thermal radiation using a hyperbolic heat flux equation, Eur.Phys.Jr.plus.,pp132-158.

- [35] SajjadHussain (2016)., Unsteady MHD stagnation flow and heat transfer for micropolar fluid towards a permeable cylinder with moving Boundaries, Science international, vol 28, No 2, pp 1967-1974.
- [36] Shamila Khalid.,(2017) Flow of MHD Thermal Stagnation point flow of Micropolar fluid due to permeable stretching surface., The Experiment : 39 Issue 4 : pp 2345-2353.
- [37] Siva reddysheri, Shamshiddin.,(2015) Heat and mass transfer on a the MHD flow of micropolar fluid in the presence of Viscous dissipation and chemical reaction,ProcediaEngineering,vol 127,pp 885-892.
- [38] Srinivas.J.,Ramana

murthy.J.V.,Ali.J.Chamkha.,(2016) Analysis of entropy generation in an inclined channel flow containing two immiscible micropolar fluids using HAM, Int.Jr.of numerical methods for heat and fluid flow,vol 26,No 3/4 ,pp 1027-1049.

- [39] Vinoop Reddy.M., Viswasairam.N., (2017) Magneto Hydrodynamic convective heat and mass transfer in a micropolar fluid, Int.Jr of research in MechEngg, vol 4, Issue 1.
- [40] Xin-Huisi, LiancunZheng, Xin-XinZhang, Ying Chao (2010), "Analytic Solution to the micropolar fluid flow through a Semi-Porous channel with an expanding or contracting wall", Applied Mathematics and Mechanics, vol 31, Issue 9, pp1073-1080.